

Multi-objective optimization of water distribution system using particle swarm optimization

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Abstract: The cost of water distribution system includes cost of pipes, pumping system, civil works and pumping energy. Out of these, cost of civil works and pumping system are nearly fixed for any specific water supply project. The cost of pipes and pumping energy are variable and can be minimized by suitable selection of pipe size, material of pipes and staging of elevated service reservoir. In the present work, the cost of pipes and energy have been optimized using Particle Swarm Optimization (PSO) method for a water supply system having large pipe network. In addition to this, the effect of swarm size and different inertia weights of PSO is also studied on the optimized cost of the system.

Key words: Water supply system, Particle swarm optimization, Inertia weight function

I. Introduction

Water is a vital commodity for all living beings on earth surface next to air [15]. Therefore water supply systems are the most important public utility for safe supply of potable water. To supply the adequate amount of water at desired pressure with minimum cost is a big challenge for researchers. The pipe and energy cost involved in the water supply contribute major share of any water supply project [7]. These two cost are variable and depend on the commercial pipe sizes, pipe material available and the staging of service reservoir. Many investigators have worked on the minimization of pipe cost taking constant height of reservoir. In this process, the pipes sizes selected for optimization may not be hydraulically efficient. The pipes selected may be either oversized or undersized to give minimum cost. Various deterministic as well as stochastic methods have been used for optimization of water distribution system [3]. Literature reveals that stochastic method are faster and gives good results for optimizing water distribution system.[1]

Particle swarm optimization is one of the best stochastic techniques for optimizing water distribution system as it has very simple features and has very fast rate of convergence. It is developed by James Kennedy and Russell Eberhart in 1995[2]. The inertia weighted function 'w' is very important parameter in PSO [5, 6]. In the present work, height of service reservoir is also minimized along with pipe cost after putting constraints on hydraulic gradient. It is seen that most of optimization of pipe network using PSO has been done using a single inertia weight function. The effect of different inertia weights on network optimization has also been presented in this paper. A computer program has been developed for the optimisation of network and analysis has been done by finite element method. The code developed is validated with existing optimised network given in literature with fixed tank staging and results are closely matching.

II. Problem formulation

The optimisation has been achieved by minimisation of pipe cost as well as energy cost by minimising height of service reservoir using following two objective functions:

2.1 Minimization of pipe cost:

The commercially available ductile iron pipes are used in design of network and objective function for minimization of pipe cost is:

$$\text{Min } Z_1 = \sum_{i=1}^{i=m} C(L_i, D_i) \quad (1)$$

2.2 Minimization of energy cost:

The pumping energy required to lift water to service reservoir depends on height of service reservoir for specific flow rate and annual energy cost is given by

$$MinZ_2 = 365 \gamma QY \eta TE_p / 1000 \tag{2}$$

In this equation, the energy cost minimized by optimizing the staging of service reservoir 'Y'. The present worth of optimized energy cost for the design period of 20 years is computed using annuity method as:

$$Z_3 = \frac{Z_2 (1+r)^t - 1}{r (1+r)^t} \tag{3}$$

2.3 Constraints and bounds:

2.3.1 Constraint 1: Diameter constraint

The commercial pipe sizes are to be used in network design optimization and the diameter chosen for design of network must be commercially available. Hence

$$D_j \in [D_k] \tag{4}$$

where D_k is the diameter of commercial available pipe set.

2.3.2 Constraint 2: Head constraint

Head at each junction must be greater than the minimum head required at each junction.

$$H_k \geq H_{min} \tag{5}$$

2.3.3 Constraint 3: Reservoir height

The staging of elevated reservoir must be within the minimum and maximum specified heights of reservoir

$$Y_{min} \leq Y \leq Y_{max} \tag{6}$$

III. Partical swarm optimization

Partical swarm optimization is Meta heuristic technique for optimization. It is developed by James Kennedy and Russell Eberhart in 1995 [10]. After each iteration, the objective function is evaluated and pbest and gbest are updated to move towards optimal solution.

In this method the initially swarm sizes are generated randomly. If the initial position of the particle is $x_i(t)$, then after the next iteration it will move to the next position of $x_i(t+1)$. The particle moves toward the best optimal solution using velocity update from $v_i(t)$ to $v_i(t+1)$ as in equation 7 and equation 8.[8].

$$v_i(t+1) = w*v_i(t) + C1*R1*(pbest - x_i(t)) + C2*R2*(gbest - x_i(t)) \tag{7}$$

$$x_i(t+1) = x_i(t) + v_i(t+1) \tag{8}$$

Where C1 and C2 are the positive constants termed as cognitive learning rate and social learning rate respectively and accelerate the particle towards the optimal solution. It is found from the literature that C1=C2=2 gives the best results for optimization and same is taken for in present work. R1 and R2 are the uniform random number ranging from 0 to 1. 'pbest' is the best solution obtained by the individual particle and 'gbest' is the best value of objective function from the entire swarm size. 'w' is the inertia weight function.[4] The different form of inertial weight functions used in PSO are tabulated in Table -1.

Table 1: Different inertia weight functions of PSO [6,11]

S. No.	Inertia weight function	Function Equation
1	Constant inertia weight	$\omega_1 = .7$
2	Random inertia weight	$w_1 = .5 + \frac{rand(1)}{2}$
3	Linear decreasing inertia weight	$w_1 = w_{max} - \left[\frac{(w_{max} - w_{min})}{max\ iteration} \right] \times iteration$
4	Logarithmic inertia weight	$w_1 = .5 \times \left(1 + \frac{1}{1 + \log(iteration)} \right)$
5.	Natural exponent inertia weight strategy e1	$w_1 = w_{min} + (w_{max} - w_{min}) \cdot e^{-\left[\frac{iter}{\frac{max\ iteration}{10}} \right]}$
6	Natural exponent inertia weight strategy e2	$w_1 = w_{min} + (w_{max} - w_{min}) \cdot e^{-\left[\frac{iter}{\frac{max\ iteration}{4}} \right]^2}$
7.	Simulated annealing inertia weight	$w_1 = w_{min} + (w_{max} - w_{min}) \times \lambda^{(iter - 1)}$ Here $\lambda = 0.95$
8.	Time varying inertia weight	$w_1 = [(w_{max} - w_{min}) \times \frac{(max\ iteration - iter)}{max\ iteration}] + w_{min}$

IV. Network design data

A residential colony developed in Bhopal, Madhya Pradesh, India with plot size 1000-1200 sqft. is taken for optimization. There is 3 floor building on each plot. The demand for network has been computed for 4 people per floor. The per capita demand per day is taken 150 liters. The ductile iron pipes are to be used for network. The layout of pipes is shown in fig.1. The complete network consists of 107 pipes and 75 nodes. The minimum head to be maintained at each junction is 17 m. Total supply hour is taken as 6 hours and pumping is done for 16 hours. The total length of pipes of network is 3796 m.

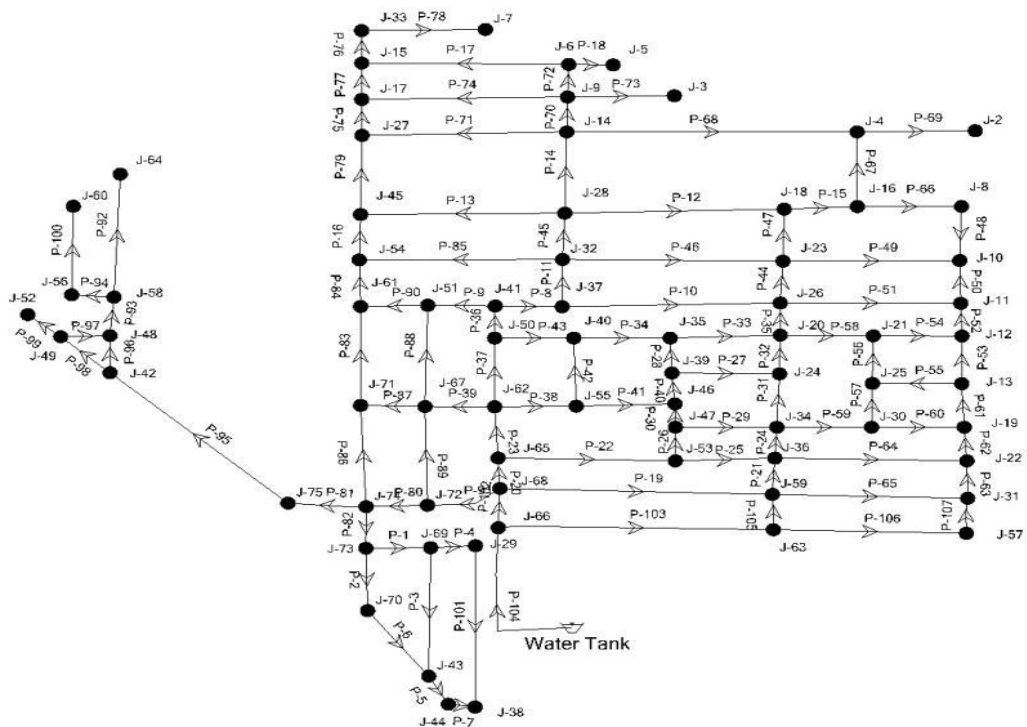


Fig1: Water distribution network .

V. Computational procedure

- i. Input network parameters like pipe length, junction demand and elevation, commercial pipe sizes, cost, design period and rate of interest, unit energy cost.
- ii. Choose the swarm size and generation random number for diameter of pipes between given range of commercially available pipe diameter set.
- iii. Replace the random number to nearest commercial pipe diameter by considering the permissible hydraulic gradient.
- iv. Carry out network analysis by finite element method.
- v. Find out pbest, gbest and fitness cost.
- vi. Update the tank height.
- vii. Update diameter set.
- viii. Repeat steps (iii) to (viii) till the solution converges to specified accuracy for pbest, gbest and reservoir staging for all inertia weights of PSO and gives same optimized cost for at least 60 iterations.

The commercial ductile iron pipes used in present network optimization are 100mm, 125mm, 150mm, 200mm, 250mm, 300mm, 350mm, 400mm, 450mm, 500mm and their corresponding unit costs are Rs.775, Rs.948, Rs.1120, Rs.1550, Rs.2100, Rs.2900, Rs.2900, Rs.3445, Rs.4015, Rs.4853, 5753[13]. The design period, rate of interest and unit energy cost are taken as 20 years, 10 % and Rs.5 respectively.

VI. Result and discussions

The analysis has been carried out for 6 swarm size ranging from 150 to 250 at interval of 20 for 8 different inertia weight function of PSO. The optimized cost of pipes, energy and sum of these two cost as total cost are presented in form of bar chart for different swarm size in fig.2 to fig.7.

It is observed from the minimum cost obtained with different swarm size and inertia weight functions that the ratio of average energy cost to pipe cost is about 45:35. It indicates that energy cost minimization is more important for economic design of pipe network however the energy cost is also dependent on the energy charges and rate of interest and design period.

It is seen from the cost variation in fig.2 to fig.7 that minimum energy cost is nearly same and found to be independent of weight function and swarm size. The lowest value of minimum cost achieved is Rs.45.01 lakhs for 5

swarm size and weight functions and highest value is Rs.45.31 lakhs at swam size 170 (fig.3) and constant weight functions. The maximum difference between the optimized energy cost from different inertia weight functions and swarm size is 8.12% at swarm size 170 while minimum difference is 0.29% at swam size 190. The overall standard deviation is 2.93

There is large variation in the minimum pipe cost obtained from optimization for different swarm size and weight functions. The highest value of optimized pipe cost is Rs.38.76 lakhs at 210 swarm size for logarithmic weight function (fig.5) while lowest cost is Rs.34.10 lakhs at swarm size 170 and random weight function (fig.3). The maximum difference between highest and lowest minimum cost is 9.21% at swarm size 210 and minimum difference is 3.82% at swarm size 230 with overall standard deviation of 2.07.

The minimum optimized total cost is again achieved at swarm size and weight functions other than where minimum optimized pipe and energy cost is obtained. The highest value of minimum cost is Rs.83.95 lakhs for logarithmic weight function at swam size 170, while lowest value is Rs.79.85 lakhs for natural exponent e-1 weight function at same swarm size 170 (Fig.3). The percentage variations in highest and lowest optimized cost at different weight function and swarm size are 5.13 % and 1.49 % respectively. The standard deviation of the percentage cost variation is 1.33.

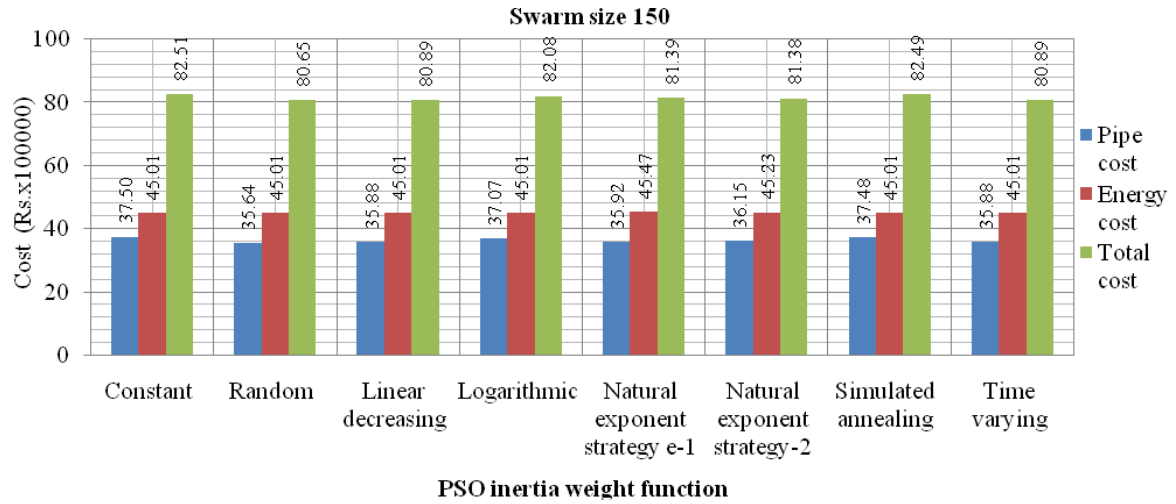


Fig 2: Variation of costs for different variant at swarm size 150

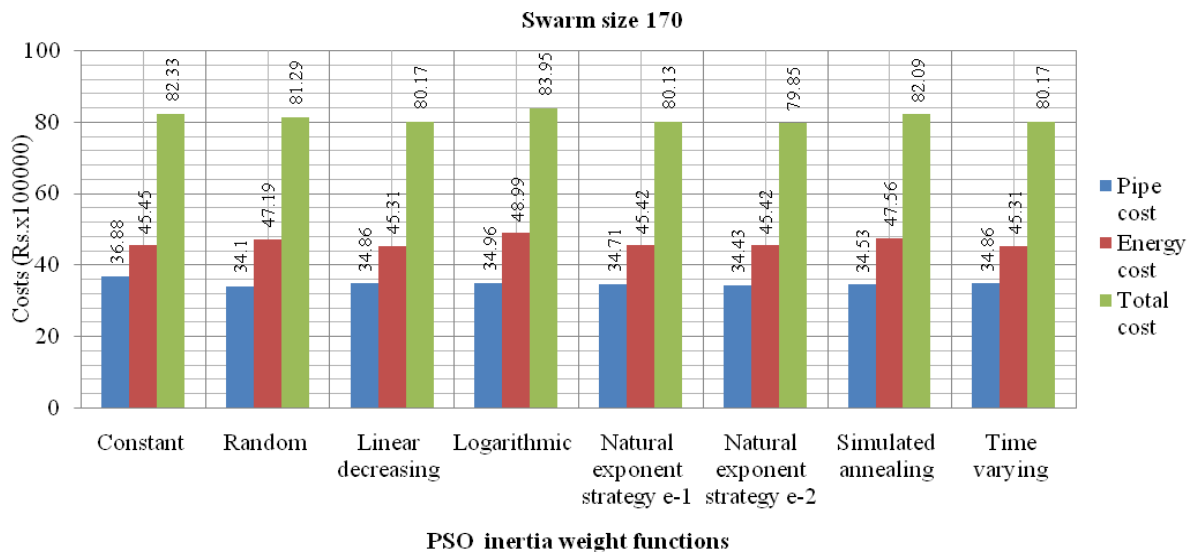


Fig 3: variation of costs for different variant at swarm size 170

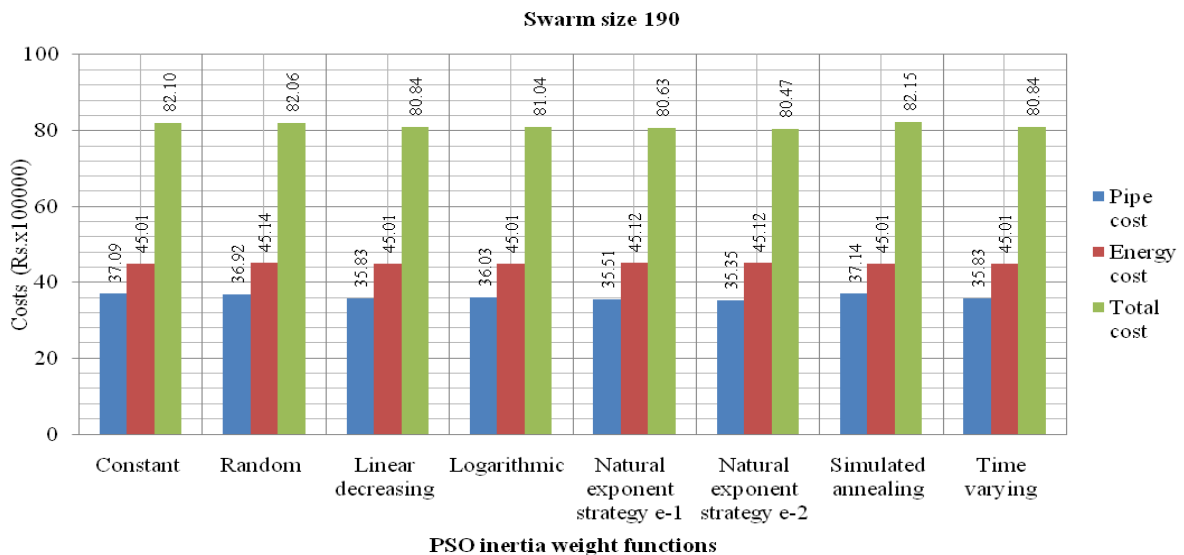


Fig. 4: Variation of costs for different variant at swarm size 190

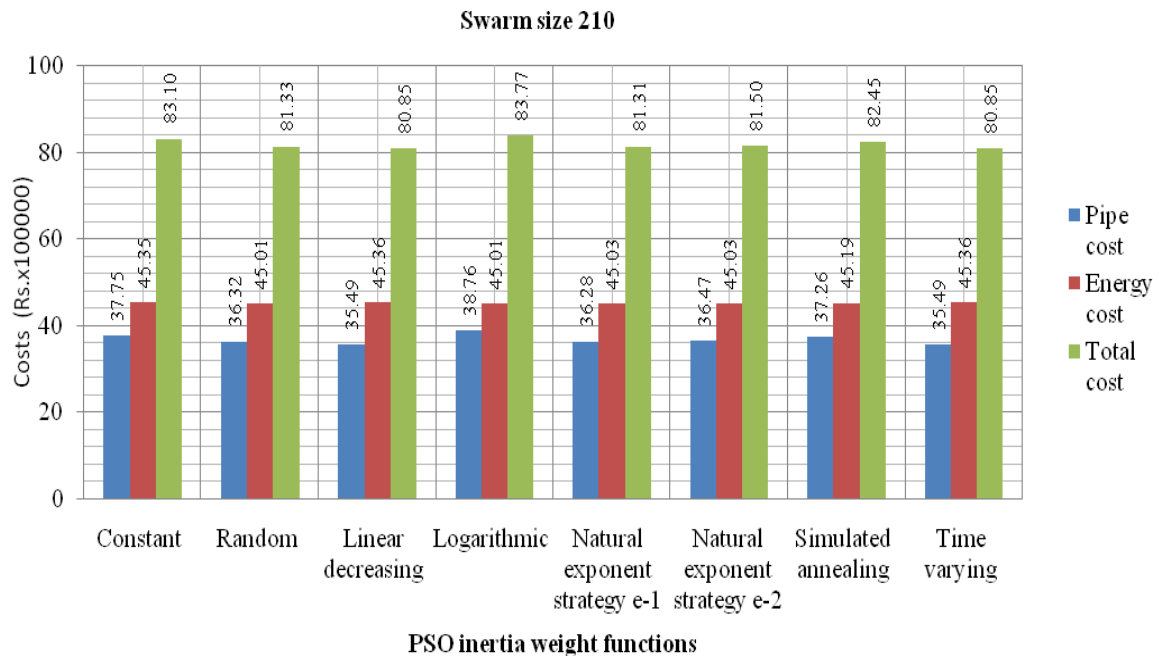


Fig. 5: Variation of costs for different variant at swarm size 210

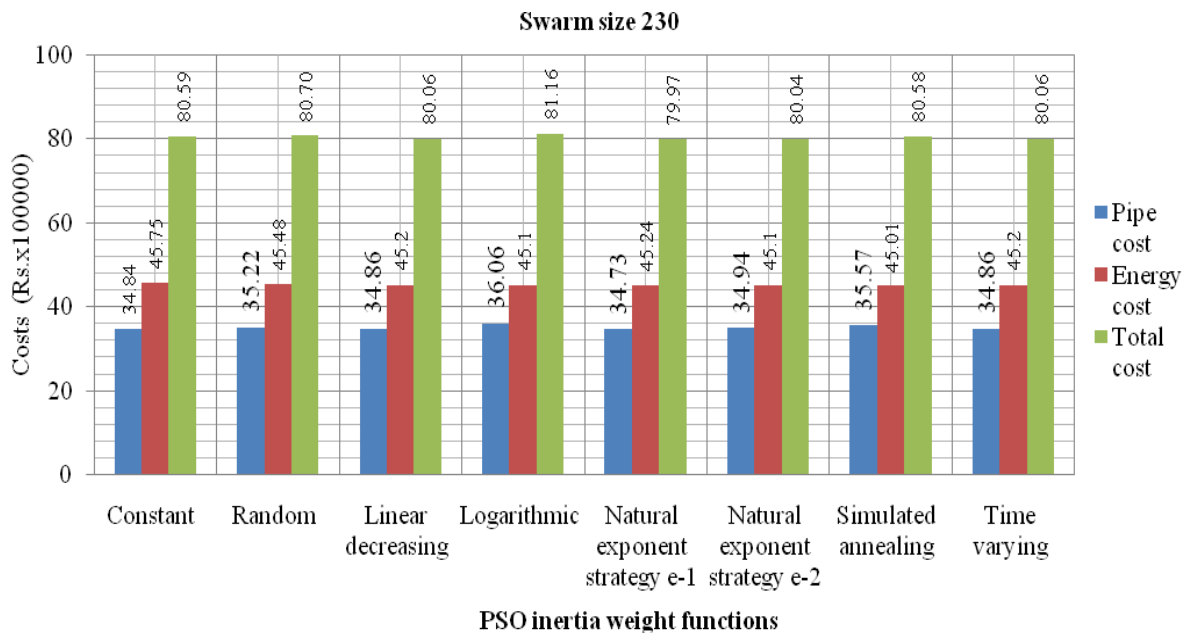


Fig. 6: Variation of costs for different variant at swarm size 230

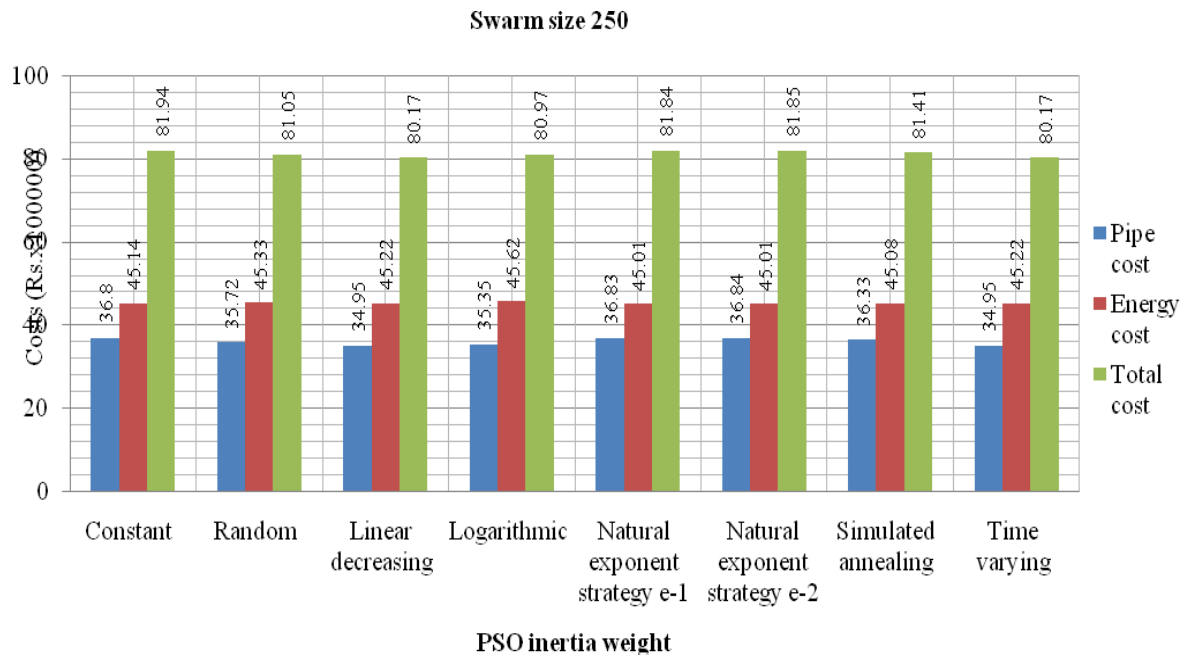


Fig. 7: Variation of costs for different variant at swarm size 250

Table 2: Highest and lowest values of optimised cost at different inertia weight functions

Inertia weight function	Energy cost (Rs. in Lakhs)			Pipe cost (Rs. in Lakhs)			Total cost (Rs. in Lakhs)		
	Highest	Lowest	Difference (%)	Highest	Lowest	Difference (%)	Highest	Lowest	Difference (%)
Constant	45.75	45.01	1.64	37.89	34.84	8.75	83.1	80.59	3.11
Random	47.19	45.01	4.84	36.92	34.10	8.27	82.06	80.65	1.75
Linear decreasing	45.36	45.01	0.78	35.88	34.86	2.93	80.86	80.07	0.99
Logarithmic	48.99	45.01	8.84	39.47	34.96	12.90	83.95	80.98	3.67
Natural exponent strategy e-1	45.47	45.01	1.02	36.28	34.71	4.52	81.84	79.98	2.33
Natural exponent strategy e-2	45.42	45.01	0.91	36.47	34.43	5.93	81.85	79.85	2.50
Simulated annealing	47.56	45.01	5.67	39.77	34.53	15.18	82.49	80.58	2.37
Time varying	45.36	45.01	0.78	35.88	34.86	2.93	80.89	80.07	1.02

Table 3: Highest and lowest values of optimised cost at different swarm size

Swarm size	Energy cost (Rs. in Lakhs)			Pipe cost (Rs. in Lakhs)			Total cost (Rs. in Lakhs)		
	Highest	Lowest	Difference (%)	Highest	Lowest	Difference (%)	Highest	Lowest	Difference (%)
150	45.47	45.01	1.02	37.50	35.64	5.21	82.51	80.65	2.31
170	48.99	45.31	8.12	36.88	34.10	8.15	83.95	79.85	5.13
190	45.14	45.01	0.29	37.14	35.35	5.06	82.15	80.47	2.09
210	45.36	45.01	0.78	38.76	35.49	9.21	83.77	80.85	3.61
230	45.75	45.01	1.64	36.06	34.73	3.82	81.16	79.97	1.49
250	45.62	45.01	1.36	36.83	34.95	5.37	81.94	80.17	2.21
Standard deviation	2.93			2.07			1.33		

VII. Conclusions

It is observed from the results of optimization of pipe network that total cost and pipe cost are affected by both swarm size and PSO variant. The optimized lowest cost energy of is nearly independent of swarm size and weight function. The capitalized energy cost over the design period is more than pipe cost and affect the optimization to large extent. The minimum optimized cost of energy is Rs.45.01Lakhs at all weight function and swarm size except swarm size 170 but optimized cost of pipe is Rs.34.10 Lakhs for swarm size 170 and random weight function . The minimum optimized total cost is Rs.79.85 Lakhs at swarm size 170 and natural exponent strategy e-2 weight function. The for maximum difference of 9.21% between highest and lowest optimized cost is seen for the pipe cost while the minimum difference 0.29% is there for energy cost. The swarm size and weight function to be chosen for optimization will depend on the size of network.

Notations:

E_p	-	Unit energy cost (Rs.)
H_k	-	Head available at each junction
H_{min}	-	Minimum required head at every junction
K	-	Total no. of junctions
h_f	-	Head loss (m)
f	-	Friction factor
L	-	Length of the pipe (m)
V	-	Velocity in the pipe (m/s)
D	-	Diameter of the pipe (m)
r	-	Rate of interest
T	-	Design period (Years)
Y	-	Staging of over head tank(m)

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